Comment on triple gauge boson interactions in the non-commutative electroweak sector

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Abstract. In this comment we present an analysis of electroweak neutral triple gauge boson couplings projected out of the gauge sector of the extended non-commutative standard model. A brief overview of the current experimental situation is given.

The main purpose of this comment is to provide a set of electroweak neutral triple gauge boson (TGB) coupling constant values that are relevant for the analysis of collider physics processes. In a previous paper [1] we presented genuine new anomalous TGB interactions, which are not present in the standard model but arise within the framework of the extended non-commutative standard model (NCSM) [2], and also in the alternative approach to the NCSM given in [3].

The range of coupling constant values for all TGB interactions was not completely computed in the previous paper [1]. Since these are necessary for any further collider physics analysis of the pure electroweak sector of the extended NCSM, we are presenting a numerical analysis of all electroweak couplings in this comment. It is observed that no two of the TGB couplings vanish simultaneously in our model as a consequence of constraints coming from the values of the standard model couplings at the M_Z scale.

It is of interest for experimentalists to find TGB couplings [4–9], as such observations would certainly contribute to the discovery of physics beyond the standard model (SM). In the light of the recent OPAL Collaboration paper [9], which presents the first limits on noncommutative QED obtained from collider experiments, it is obvious that in order to repeat the same $e^+e^- \to \gamma\gamma$ analysis within the framework of the extended NCSM [1, 2], a complete set of values of neutral triple gauge boson coupling constants is necessary. The non-commutative fermion–photon and fermion–Z boson couplings have been given in [2]. Note that strictly SM forbidden decays coming solely from the gauge sector of the NCSM could also be probed in high-energy collider experiments.

There are two approaches to the construction of non-commutative generalizations of the standard model. The approach [3] uses some clever tricks to circumvent the problems of charge quantization and the restriction of the non-commutative gauge group only to $U(N)$: It starts with an enlarged gauge group $U(3) \times U(2) \times U(1)$ and then removes superfluous $U(1)$ factors with the help of extra Higgs fields (higgsacs). The hypercharges and the electric charges are still quantized but now to the correct values of the usual quarks and leptons. The other approach [2] to the NCSM solves the standard problems of non-commutative model building with the help of generalized Seiberg–Witten maps. In principle the two approaches can also be combined. Details of the latter approach of the München–Wien groups are given in [10–14]. We propose the use of effective Lagrangians constructed within the NCSM [1] in further analysis of scatterings of electrons and photons. This is the only approach that allows one to build models of the electroweak sector directly based on the structure group $U(1) \times SU(2)$ in the presence of space-time non-commutativity.

The action that we use here should be understood as an effective theory up to linear order in the non-commutativity parameter θ. New triple gauge boson terms in the action have the following form [1]:

$$S_{\text{gauge}} = -\frac{1}{4} \int \mathrm{d}^4 x f_{\mu\nu} f^{\mu\nu} \tag{1}$$

$$-\frac{1}{2} \int \mathrm{d}^4 x \, \mathrm{Tr}\,(F_{\mu\nu} F^{\mu\nu}) - \frac{1}{2} \int \mathrm{d}^4 x \, \mathrm{Tr}\,(G_{\mu\nu} G^{\mu\nu})$$

$$+ g_s \theta^{\rho\tau} \int \mathrm{d}^4 x \, \mathrm{Tr}\left(\frac{1}{4} G_{\rho\tau} G_{\mu\nu} - G_{\mu\rho} G_{\nu\tau}\right) G^{\mu\nu}$$

$$+ g'^3 \kappa_1 \theta^{\rho\tau} \int \mathrm{d}^4 x \left(\frac{1}{4} f_{\rho\tau} f_{\mu\nu} - f_{\mu\rho} f_{\nu\tau}\right) f^{\mu\nu}$$

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$$+\, g'g^2\kappa_2\theta^{\rho\tau}\int \mathrm{d}^4x \sum_{a=1}^{3}\left[\left(\frac{1}{4}f_{\rho\tau}F_{\mu\nu}^a - f_{\mu\rho}F_{\nu\tau}^a\right)F^{\mu\nu,a} + \mathrm{c.p.}\right]$$

$$+\, g'g_s^2\kappa_3\theta^{\rho\tau}\int \mathrm{d}^4x \sum_{b=1}^{8}\left[\left(\frac{1}{4}f_{\rho\tau}G_{\mu\nu}^b - f_{\mu\rho}G_{\nu\tau}^b\right)G^{\mu\nu,b} + \mathrm{c.p.}\right],$$

where c.p. means cyclic permutations. Here $f_{\mu\nu}$, $F_{\mu\nu}^a$ and $G_{\mu\nu}^b$ are the physical field strengths corresponding to the groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. The constants κ_1, κ_2 and κ_3 are functions of $1/g_i^2$ $(i=1,\ldots,6)$:

$$\kappa_1 = -\frac{1}{g_1^2} - \frac{1}{4g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} + \frac{1}{36g_5^2} + \frac{1}{4g_6^2},$$

$$\kappa_2 = -\frac{1}{4g_2^2} + \frac{1}{4g_5^2} + \frac{1}{4g_6^2},$$

$$\kappa_3 = +\frac{1}{3g_3^2} - \frac{1}{6g_4^2} + \frac{1}{6g_5^2}. \tag{2}$$

The g_i are the coupling constants of the non-commutative electroweak sector up to first order in θ. The appearance of new coupling constants beyond those of the standard model reflect a freedom in the strength of the new TGB couplings. Matching the SM action at zeroth order in θ, three consistency conditions are imposed on (1):

$$\frac{1}{g'^2} = \frac{2}{g_1^2} + \frac{1}{g_2^2} + \frac{8}{3g_3^2} + \frac{2}{3g_4^2} + \frac{1}{3g_5^2} + \frac{1}{g_6^2},$$

$$\frac{1}{g^2} = \frac{1}{g_2^2} + \frac{3}{g_5^2} + \frac{1}{g_6^2},$$

$$\frac{1}{g_s^2} = \frac{1}{g_3^2} + \frac{1}{g_4^2} + \frac{2}{g_5^2}. \tag{3}$$

From the action (1) we extract the genuine new neutral triple gauge boson terms which are not present in the SM Lagrangian. In terms of the SM physical fields G, A and Z, they are [1]

$$\mathcal{L}_{\gamma\gamma\gamma} = \frac{e}{4}\sin 2\theta_W K_{\gamma\gamma\gamma}\theta^{\rho\tau}A^{\mu\nu}\left(A_{\mu\nu}A_{\rho\tau} - 4A_{\mu\rho}A_{\nu\tau}\right),$$

$$K_{\gamma\gamma\gamma} = \frac{1}{2}gg'(\kappa_1 + 3\kappa_2);$$

$$\mathcal{L}_{Z\gamma\gamma} = \frac{e}{4}\sin 2\theta_W K_{Z\gamma\gamma}\theta^{\rho\tau}\left[2Z^{\mu\nu}\left(2A_{\mu\rho}A_{\nu\tau} - A_{\mu\nu}A_{\rho\tau}\right)\right.$$
$$\left. +\, 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau} - Z_{\rho\tau}A_{\mu\nu}A^{\mu\nu}\right],$$

$$K_{Z\gamma\gamma} = \frac{1}{2}\left[g'^2\kappa_1 + \left(g'^2 - 2g^2\right)\kappa_2\right];$$

$$\mathcal{L}_{ZZ\gamma} = \mathcal{L}_{Z\gamma\gamma}(A \leftrightarrow Z),$$

$$K_{ZZ\gamma} = \frac{-1}{2gg'}\left[g'^4\kappa_1 + g^2\left(g^2 - 2g'^2\right)\kappa_2\right];$$

$$\mathcal{L}_{ZZZ} = \mathcal{L}_{\gamma\gamma\gamma}(A \to Z),$$

$$K_{ZZZ} = \frac{-1}{2g^2}\left[g'^4\kappa_1 + 3g^4\kappa_2\right];$$

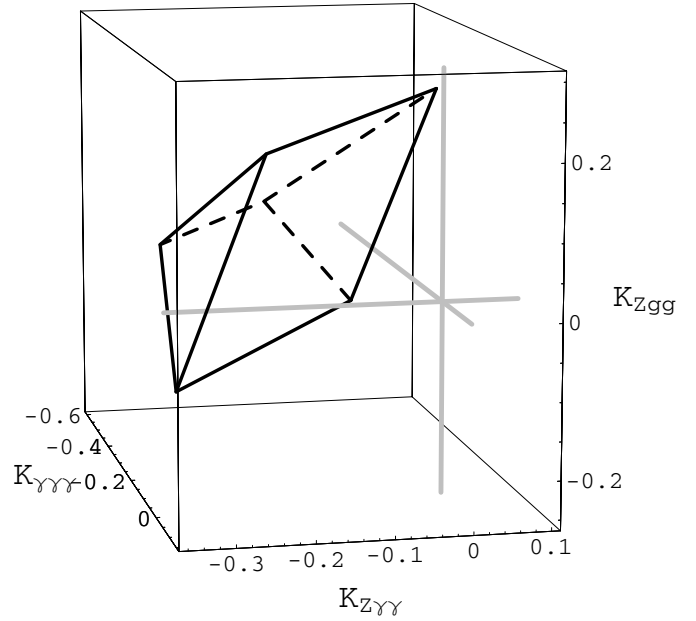$$\mathcal{L}_{Zgg} = \mathcal{L}_{Z\gamma\gamma}(A \to G^b),$$

Fig. 1. The three-dimensional pentahedron that bounds possible values for the coupling constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$ and K_{Zgg} at the M_Z scale

$$K_{Zgg} = \frac{g_s^2}{2}\left[1 + \left(\frac{g'}{g}\right)^2\right]\kappa_3;$$

$$\mathcal{L}_{\gamma gg} = \mathcal{L}_{Zgg}(Z \to A),$$

$$K_{\gamma gg} = \frac{-g_s^2}{2}\left[\frac{g}{g'} + \frac{g'}{g}\right]\kappa_3, \tag{4}$$

where $A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, etc.

The above three conditions (3), together with the requirement that $1/g_i^2 > 0$, define a three-dimensional pentahedron in the six-dimensional moduli space spanned by $1/g_1^2, \ldots, 1/g_6^2$. It is possible to express all constraints and conditions in terms of coupling constants $K_{\gamma\gamma\gamma}$, etc. Figure 1 shows the three-dimensional pentahedron [1] that bounds the allowed values for the dimensionless coupling constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$ and K_{Zgg}, at the scale M_Z. For any chosen point within the pentahedron in Fig. 1, the remaining three coupling constants $K_{ZZ\gamma}$, K_{ZZZ} and $K_{\gamma gg}$, are uniquely fixed by the equations

$$K_{ZZ\gamma} = \frac{1}{2}\left(\frac{g}{g'} - 3\frac{g'}{g}\right)K_{Z\gamma\gamma} - \frac{1}{2}\left(1 - \frac{g'^2}{g^2}\right)K_{\gamma\gamma\gamma},$$

$$K_{ZZZ} = \frac{3}{2}\left(1 - \frac{g'^2}{g^2}\right)K_{Z\gamma\gamma} - \frac{1}{2}\frac{g'^2}{g^2}\left(3 - \frac{g'^2}{g^2}\right)K_{\gamma\gamma\gamma},$$

$$K_{\gamma gg} = -\frac{g}{g'}K_{Zgg}. \tag{5}$$

The values for all six coupling constants at the pentahedron vertices are given in Table 1. From Table 1 it is possible to construct the allowed region for any pair of couplings: $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$, $K_{ZZ\gamma}$, K_{ZZZ}, $K_{\gamma gg}$ and $K_{Z\gamma\gamma}$. The range of values for a full set of electroweak coupling constants is given in Figs. 2 to 7.

Table 1. The values of the triple gauge boson couplings at the vertices of the pentahedron in the extended NCSM at the M_Z scale

$K_{\gamma\gamma}$	$K_{Z\gamma\gamma}$	K_{Zgg}	$K_{ZZ\gamma}$	K_{ZZZ}	$K_{\gamma gg}$
-0.184	-0.333	0.054	0.035	-0.213	-0.098
-0.027	-0.340	-0.108	-0.021	-0.337	0.197
0.129	-0.254	0.217	-0.068	-0.362	-0.396
-0.576	0.010	-0.108	0.202	0.437	0.197
-0.497	-0.133	0.054	0.162	0.228	-0.098
-0.419	0.095	0.217	0.155	0.410	-0.396

Fig. 5. The allowed region for $(K_{ZZ\gamma}, K_{\gamma\gamma})$

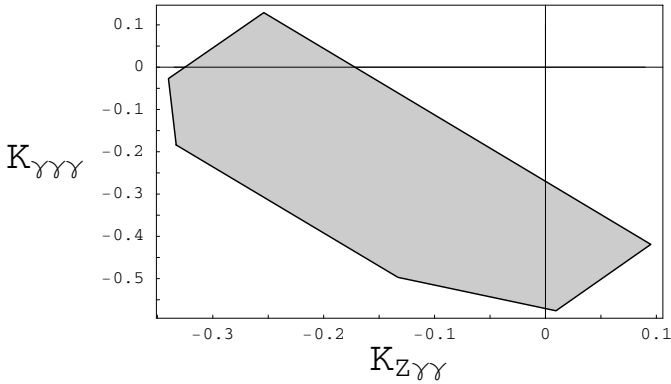**Fig. 2.** The allowed region for $(K_{Z\gamma\gamma}, K_{\gamma\gamma})$ couplings

Fig. 6. The allowed region for $(K_{ZZZ}, K_{\gamma\gamma})$
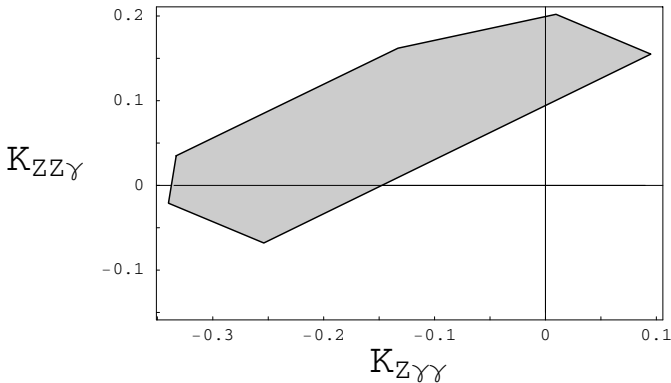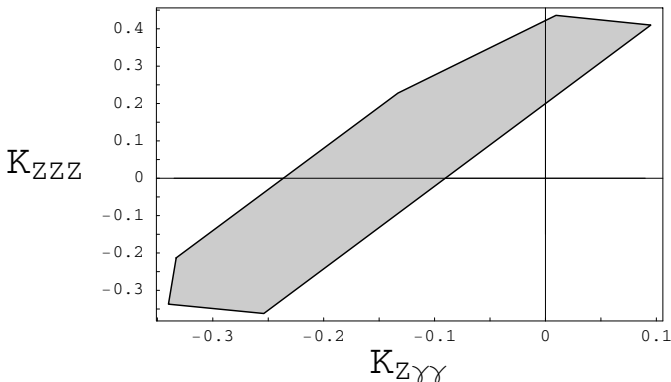

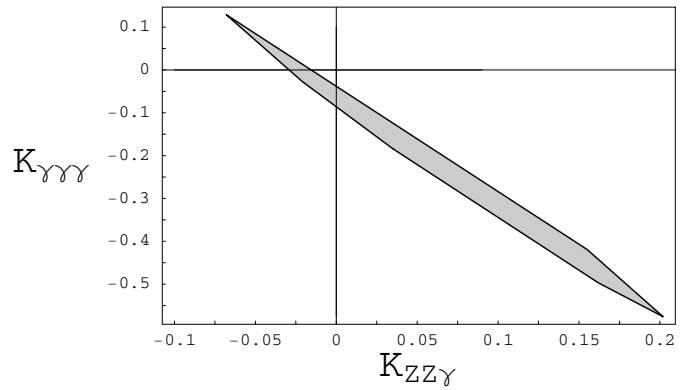**Fig. 3.** The allowed region for $(K_{Z\gamma\gamma}, K_{ZZ\gamma})$

Fig. 7. The allowed region for $(K_{ZZZ}, K_{ZZ\gamma})$

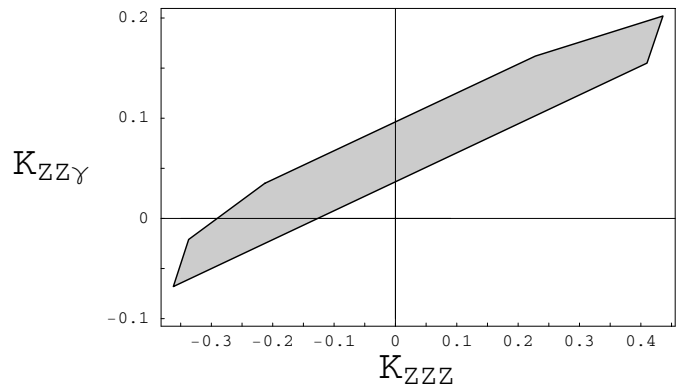**Fig. 4.** The allowed region for $(K_{Z\gamma\gamma}, K_{ZZZ})$

The important property evident from (5) and Figs. 2 to 7 is that any combination of two TGB coupling constants from the gauge sector can never vanish simultaneously due to the constraint set by the value of the SM coupling constants at the M_Z scale.

Evidence for non-commutativity coming from the gauge sector should be searched for in processes involving the above couplings. The experimental discovery of the kinematically allowed $Z \to \gamma\gamma$ decay would indicate a violation of the Yang theorem and would be a possible signal of space-time non-commutativity. This would fix the quantity $|K_{Z\gamma\gamma}/\Lambda_{\mathrm{NC}}^2|^2$, where Λ_{NC} represents the scale of non-commutativity. Inclusion of other triple gauge bo-

son interactions through $2 \rightarrow 2$ scattering experiments would sufficiently reduce the available parameter space of our model.

To get an idea about the order-of-magnitude of the rate, let us choose the central value of the $Z\gamma\gamma$ coupling constants $|K_{Z\gamma\gamma}| \simeq 1/10$ and assume that maximal non-commutativity occurs at the scale of $\sim 1\,\mathrm{TeV}$. The resulting branching ratio [1] for this decay, at tree level, would then be $\mathrm{BR}(Z \rightarrow \gamma\gamma) \simeq 4 \times 10^{-8}$.

Concerning the question of the scale of non-commutativity Λ_{NC} in forbidden decays and in scatterings of electrons and photons, the experimental situation can be summarized as follows.
(i) The joint efforts of the DELPHI, ALEPH, OPAL and L3 Collaborations [4,5,8,9] give us hope that in not too much time all collected data from the LEP experiments will be counted and analysed, producing tighter bounds on triple gauge boson couplings and the scale of non-commutativity, i.e. on the quantities like $|K_{Z\gamma\gamma}/\Lambda_{\mathrm{NC}}^2|^2$, etc.
(ii) The sensitivity to the NC parameter $\theta^{\mu\nu}$ could be in the range of the next generation of linear colliders, with a CM energy around a few TeV.
(iii) The first limits on the NCQED obtained from collider experiments were recently presented by the OPAL Collaboration in [9]. They found no significant deviation from the SM prediction and at the 95% confidence level the limit was set on the non-commutative scale $\Lambda_{\mathrm{NC}} > 141\,\mathrm{GeV}$. This is valid for all relevant angles that determine a unique direction in space.
(iv) Finally, note that the best testing ground for studies of anomalous TGB couplings, before the start of the linear e^+e^- collider, will be the LHC [16].

In conclusion, the gauge sector of the $SU(2) \times U(1)$ is a possible place for the experimental discovery of space-time non-commutativity. We believe that the importance of a possible discovery of space-time non-commutativity at very short distances would convince collider particle physics experimentalists to search further for the forbidden decay $Z \rightarrow \gamma\gamma$, as well as for other anomalous triple neutral gauge boson couplings.

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